Q: How do I measure "Parallax?"

A: Here are two methods:

First let's examine what causes Parallax.

The cause of parallax is simple: The images of the scope reticle and the distant object appear at different (diopter) distances. When you move your eye back and forth in the scope exit pupil, these images shift relative to each other. Illustrations 1 and 2 depict the situation for scopes with and without parallax.

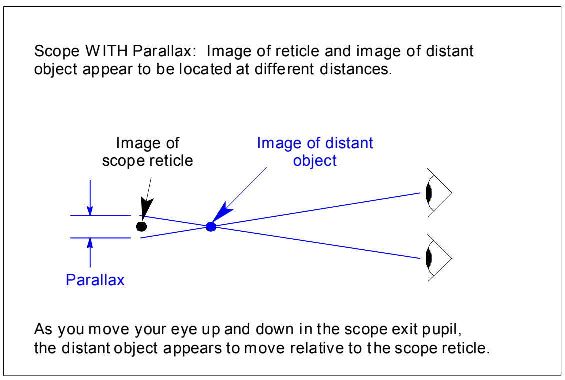


*Illustration 1: Scope **with Parallax***

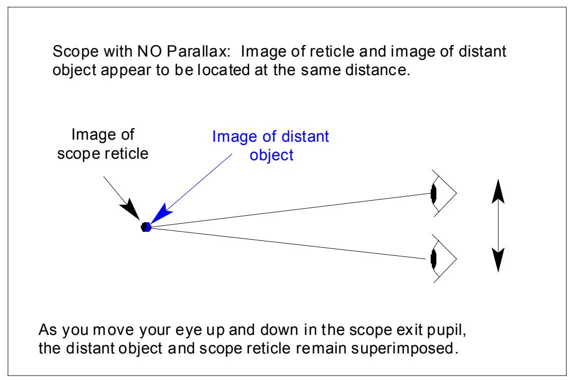


*Illustration 2: Scope with **no parallax***

Our eyes aren't very good at measuring diopter distance, but they are very good at noticing small motions between nearby objects. In other words, even though the *cause* of parallax is a distance disparity, almost everybody *experiences* parallax as a lateral shift.

How can we measure parallax?

I: The first method: Simply move your eye (or your video telescope) back and forth in the exit pupil of the scope. If you are making the measurement on the range, you can estimate the shift using tic marks on the scope reticle. If you are using an optical bench you can measure the image shift very accurately with software.

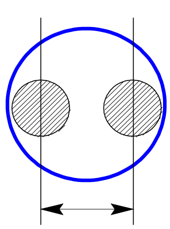


Illustration 3: Motion of telescope aperture

The blue circle represents the exit pupil of the scope. A typical value for a low power scope might be 12 mm diameter. For a high power scope the exit pupil would be much smaller.

The shaded circles represent motion of the observers eye to different positions in the exit pupil. When using an otpical bench it is common to use a 4 mm aperture to simulate the human eye pupil.

The video telescope is moved to two locations in the exit pupil, and at each position the relative position of the reticle and the distant target is measured. Appendix A explains this process in more detail.

II: The second method is based on measuring the image distance of the reticle and target in diopters. This is a little less intuitive, but is actually much faster in practice and is also much more precise. <u>We recommend you use the second method.</u>

Concept:
If we know the location of the scope reticle and target image (in diopters) then we can calculate the parallax.

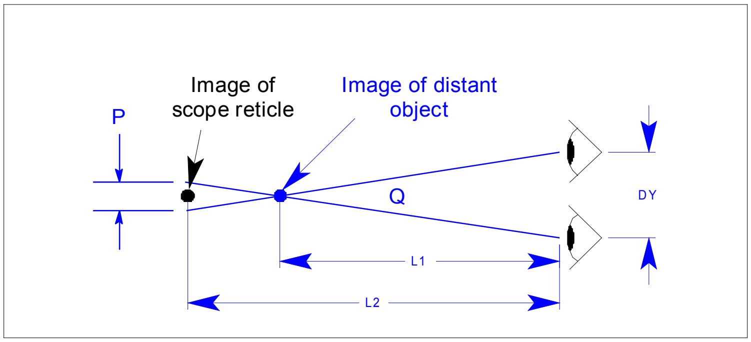


Illustration 4: Calculating Parallax "P" from distances L1 and L2

The calculation is not too complicated[1]. However, in practice it's easiest to use a spreadsheet[2]:

This spreadsheet calculates Parallax
You must enter the blue values. The spreadsheet will calculate red values

given values		Units
Magnification	**10**	X
Distant object location	**0.125**	diopters
Scope reticle location	**0.02**	diopters
Exit pupil diameter	**6**	mm
Intermediate results		
*L1_*	*8*	*meters*
*L2_*	*50*	*meters*
DY (in meters)	*0.006*	*meters*
Parallax at eyepiece	*0.630*	*Milli-radians*
Final results		
parallax at 100 yards	**0.23**	**inches**
Parallax at object	**0.063**	**Mils (milli-radians)**

- **Magnification**: If you require a really precise parallax value then you should use the measured magnification value of the scope. Otherwise the nominal magnification is fine.

1 Simple geometry will allow you to calculate the amount of linear parallax P in figure 4. However, you probably want to know the amount of Parallax at 100 yards, not at the arbitrary distance L2. The spreadsheet calculates this for you.

2 Contact Wells Research if you would like a copy of this spreadsheet.

- **Distant object location, Scope reticle location**: There are two ways to obtain these values:
  - Visually focus using the jog buttons on the motor control form. Once you have found the sharpest image location, enter the diopter values directly from the motor control form.
  - Use the "MTF vs Focus" measurement to find the precise diopter values.

- **"Shift within exit pupil"** In Illustration 4 this is labeled "DY" Obviously the shift can't be larger than the pupil diameter, but it can be smaller. Unless you have a specific reason to use a different value we suggest you enter the actual exit pupil diameter. Appendix "A" discusses this issue in more detail.

I hope this FAQ has been helpful. If you have comments or questions please send them to me at info@wellsresearch.com.

Ben Wells
May 1, 2014

Appendix A:

This section explores the following question: How much should you move the video telescope when measuring parallax?

Some examples:

The blue circle represents the scope exit pupil (let's say it's 12 mm diameter) The black shaded circles represent the aperture of the measuring telescope (let's say it is 4 mm diameter.)

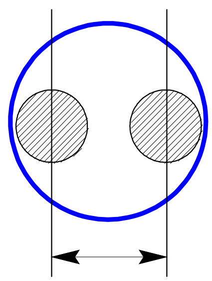


Illustration 5: Sampling aperture moved by 8 mm

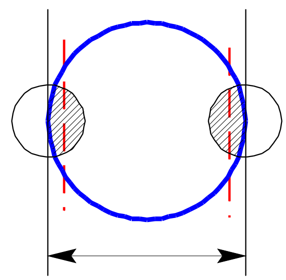


Illustration 6: Sampling aperture physically moved by 12 mm., although effective shift is less.

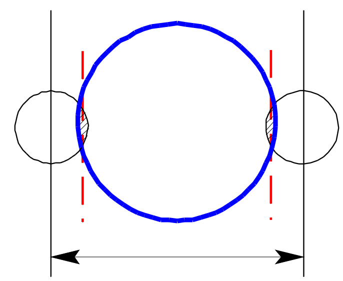


Illustration 7: worst case shift is almost the full pupil diameter

- In illustration 5 I've moved the sampling aperture by 8 mm. Is this where I should measure parallax?

  Well, if I were making measurements on a scope made by my own company, I might argue that this was the correct amount of shift. After all, if I shift more, (illustration 6) I'll experience some loss of brightness in the images.

- And if I were measuring a competitor's scope? Ah well <u>then</u> I might argue that I should use all the motion I could, since this would give a high number. I might argue that illustration 6, or even illustration 7 was the correct amount to shift.

Some takeaway points:

- The more you shift your eye, the more parallax you see.

- The worst-case is shown in Illustration 7. However, its important to remember this is a limit condition. In other words, the closer you get to the edge, the less light that reaches your eye (or your measurement telescope). You can measure close to the cutoff point, but not actually at it.

- I'm not aware of any industry consensus on whether the “right” amount of shift is shown in Illustration 5, 6, or 7. I personally vote for the worst-case approach in Illustration 7, but it's really up to you.

- By the way, if you feel the worst-case represented by Illustration 7 is the correct approach, <u>and</u> if you insist on measuring parallax by phsically shifting the telescope, it's still not necessary to struggle with vignetting. You can measure with the sampling aperture comfortably within the scope pupil (Illustration 5) and then apply a correction factor (in this case 1.5X) to predict what you would measure at the worst-case shown in Illustration 7.

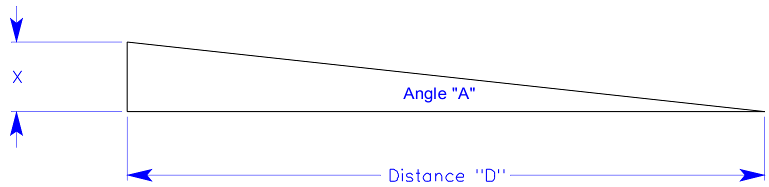
Very high power scopes:

- What if a high power scope has a very small exit pupil (say 1.5 mm). Since the human eye pupil is rarely smaller than 2 mm, is it even possible for such a scope to have parallax?
  This is pretty theoretical, but it's worth discussing. The answer is "yes" for two reasons.
  - First, refer back to Illustration 1. The fundamental cause of parallax is a focus discrepancy, A focus discrepancy is absolutely possible even if the exit pupil is small.
  - Even if the scope exit pupil is very small, it's still possible to sample it selectively by moving the eye (or telescope) back and forth near the edge.

Appendix B

How do you convert an *angular* measurement of parallax (measured at the eyepiece) to *linear* units at a distant target, for example inches at 100 yards?

(1)
The basic trig to convert an angle to a distance is simple:



X = D Tan(A)

If you use a spreadsheet like Excel you have to rembember[3] to specify A in units of Radians.

Example 1:	A= 1 milli-radian	D = 100 yards	X= 3.6 inches
Example 2:	A= 1 degree	D=100 yards	X= 62.8 inches

(2)
Don't forget to include the magnification of the scope:

If the scope has 40 power, then moving the target 1/40 degree will look like a 1 degree motion when you look in the eyepiece. If you want to calculate parallax is linear distance like inches then the correction is simple: Before you make the trig calculation, divide by the scope power.

Example 3:

A(eye) = 1 degree	A(targ) = 1/40 degree	D= 100 yards	X= 1.57 inches

(3)
Mils (milli-radians)
It's a little more confusing when discussing angular units, but the result is the same: You must divide the measured value by the scope power.

When people say a scope has 1 mil of parallax, they mean that the parallax is 3.6 inches at 100 yards.

For example, if you found a 40X scope had 4 mils of parallax measured <u>at the eyepiece</u>, users would understand this best if you described this as 0.1 mil of parallax at 100 yards[4].

3 You divide by 57.3 (or 57.29578 if you love lots of decimal places)
For example 1 degree = 1/57.3 = 0.0174 Rad = 17.4 mils (milli-radians)

4 The observant reader may say "why do you say 'mils at 100 yards?' Isn't a mil an angular measurement that is independent of distance? " (1) Yes, it is (2) Convention in the industry is strong. People will understand you better if you go with the flow and say "mils at 100 yards"

Appendix C:

This section discusses how to measure parallax by moving the video telescope to different positions in the exit pupil.

We actually recommend a different method, using diopter measurement + spreadsheet, but this section may be of interest for reference

- First, how much should we move the video telescope? This seems like a simple enough question, with an obvious answer. However, it's worth exploring this in more depth (see appendix A, and also footnote[2] below)

- The actual calculation is straightforward: We use the video telescope to make measurements at two locations within the exit pupil:
  - Angular location of image of reticle: Lets call these angular locations R1 and R2.
  - Angular location of image of distant target: Call these angular locations O1 and O2.

  Parallax in eyepiece= [(R1-O1) - (R2-O2)] • scale_factor

  See footnote[5] for discussion of scale_factor

- The process above measures the parallax in angular units like degrees or mils (milli-radians) at the eyepiece. Scope users are more interested in the amount of parallax at the target. Appendix B explains how to make this conversion.

  It's customary to state the parallax in linear units like inches at 100 yards. Appendix B also exlains how to convert to linear units.

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5 On the range a user would probably evaluate parallax by moving back and forth between the extremes of the exit pupil (see Illustration 7).

On the bench this isn't practical: If you truly move to the edge of the pupil (illustration 7) then little or no light enters the video telescope! Instead, it's better to move a shorter distance (as suggested in illustration 6), and the apply a scale factor. The scale factor corrects for the fact that you are only moving partway across the pupil.